JHEP HYPER VERSION

Received: *November 12, 2001*, Accepted: *November 20, 2001*

The Ramond sector of open superstring field theory

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Abstract: Although the equations of motion for the Neveu-Schwarz (NS) and Ramond (R) sectors of open superstring field theory can be covariantly expressed in terms of one NS and one R string field, picture-changing problems prevent the construction of an action involving these two string fields. However, a consistent action can be constructed by dividing the NS and R states into three string fields which are real, chiral and antichiral. The open superstring field theory action includes a WZW-like term for the real field and holomorphic Chern-Simons-like terms for the chiral and antichiral fields. Different versions of the action can be constructed with either manifest $d = 8$ Lorentz covariance or manifest $N = 1$ $d = 4$ super-Poincaré covariance. The lack of a manifestly $d = 10$ Lorentz covariant action is related to the self-dual five-form in the type-IIB R-R sector.

Keywords: Superstrings and Heterotic Strings, BRST Symmetry.

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1. Introduction

Open superstring field theory has recently returned to center stage due to the work of Sen and others on tachyon condensation [1]. Although the Ramond sector of open superstring field theory is not directly related to tachyon condensation, an understanding of this sector is crucial for studying other properties of the superstring action. For example, it will be argued below that certain terms involving the Ramond sector are expected to satisfy non-renormalization theorems.

Although a cubic open superstring field theory action involving both the Neveu-Schwarz (NS) and Ramond (R) sectors was proposed in [2], this action (as well as all others [3] constructed using picture-changing operators) suffers from problems caused by the presence of picture-raising and picture-lowering operators. The picture-raising operator appears in the NS interaction term and leads to contact-term divergences in tree-level scattering amplitudes [4]. The picture-lowering operator appears in the R kinetic term and, as will be discussed in section 2, leads to a breakdown of gauge invariance due to its nontrivial kernel. Another problem with the picture-lowering operator is that it does not commute with the b ghost and therefore cannot appear in the closed superstring kinetic term for the R-NS, NS-R or R-R sectors.

As discussed in earlier papers [5, 6], the above picture-changing problems can be avoided by working in the large RNS Hilbert space [7] which includes the ξ zero mode coming from fermionizing the (β, γ) ghosts. The NS contribution to the field theory action resembles a Wess-Zumino-Witten model where the group generator g is related to the NS string field Φ by $g = e^{\Phi}$ and multiplication of string fields in the exponential uses Witten's midpoint interaction [8].

A natural question is how to include the Ramond contribution to the superstring field theory action. This question was partially answered in [5] where a manifestly $N = 1$ $d = 4$ super-Poincaré covariant action was constructed by splitting the NS and R states into a real, chiral and antichiral string field. However, the resulting action was quite complicated and it was unclear if other actions could be constructed which preserve more symmetries.

In this paper, it will be argued that splitting the NS and R states into three string fields is necessary for constructing consistent superstring field theory actions. Although the open superstring field theory equations of motion can be covariantly expressed in terms of a single NS and R string field, it is not possible to construct a consistent action out of these two string fields.[1] But by using three string fields consisting of a real, chiral and antichiral string field, a consistent action can be constructed which reproduces the desired equations of motion. This action includes a WZW-like term constructed from the real string field, a kinetic term for the chiral and antichiral string fields coupled minimally to the real field, and a holomorphic and antiholomorphic Chern-Simons-like term constructed from the chiral and antichiral fields. The holomorphic Chern-Simons-like term involves integration over a chiral subspace and therefore resembles a superspace F-term. For the usual reasons, this F-term is expected to satisfy non-renormalization theorems.

Depending on how the NS and R states are distributed among the three string fields, different subgroups of $d = 10$ super-Poincaré covariance can be manifestly preserved. For example, in a flat background, superstring field theory actions can be constructed which manifestly preserve either $d = 8$ Lorentz covariance or $N = 1$ $d = 4$ super-Poincaré covariance. The difficulty in constructing actions with manifest $d = 4k + 2$ Lorentz covariance is related to the presence of self-dual $(2k + 1)$-forms in $d = 4k + 2$.

In section 2 of this paper, equations of motion will be defined using a single NS and R string field, and it will be argued that they cannot come from varying an action. In section 3, an open superstring field theory action will be constructed by splitting the superstring states into three string fields. In section 4, it will be shown how

[1] In fact, the superstring field theory equations of motion can also be expressed in terms of a single string superfield in a manifestly $d = 10$ super-Poincaré covariant manner [9]. However, it does not appear possible to construct an action in terms of this single string superfield.

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different choices for splitting the superstring states into three string fields produce actions with different manifest symmetries. And in section 5, some open questions will be discussed including the construction of a closed superstring field theory action.

2. Problems using two string fields

2.1 Equations of motion

Although it will not be possible to construct a consistent action using a single NS and R string field, one can define equations of motion using these two string fields. In the action of [2], the ghost-number one[2] NS and R string fields are defined in the small RNS Hilbert space and carry picture -1 and $-1/2$. For example, the massless gluon $A_m(x)$ and gluino $\chi^\alpha(x)$ are represented by the vertex operators $ce^{-\phi}\psi^m A_m(x)$ and $ce^{-\frac{1}{2}\phi}\Sigma_\alpha\chi^\alpha(x)$ where Σ_α is the RNS spin field. However, as was shown in [5], it is more convenient for constructing actions to use string fields in the large RNS Hilbert space at ghost number zero. To linearized level, these ghost number zero string fields are related to the ghost number one string fields of [2] by adding the ξ zero mode. Since ξ carries picture $+1$, it is natural to define the NS and R string fields, Φ and Ψ, to carry picture 0 and 1/2. In other words, the massless gluon and gluino will be represented by the vertex operators $\Phi = \xi ce^{-\phi}\psi^m A_m(x)$ and $\Psi = \xi ce^{-\frac{1}{2}\phi}\Sigma_\alpha\chi^\alpha(x)$.

As discussed in [10], the linearized equations of motion and gauge invariances for ghost number zero string fields in the large Hilbert space can be written

$$Q\widetilde{\eta}\Phi = Q\widetilde{\eta}\Psi = 0\,, \qquad \delta\Phi = Q\Lambda_0 + \widetilde{\eta}\Lambda_1\,, \qquad \delta\Psi = Q\Lambda_{1/2} + \widetilde{\eta}\Lambda_{3/2}\,, \tag{2.1}$$

where $Q = \int[c(T_m - b\partial c - \partial^2\phi - \frac{1}{2}(\partial\phi)^2 + \eta\partial\xi) + \eta e^\phi G_m - \eta\partial\eta e^{2\phi} b]$ is the RNS BRST operator, $\widetilde{\eta}$ denotes the zero mode of the η ghost,[3] and Λ_n are ghost-number -1 string fields of picture n. Note that the linearized gauge transformations parameterized by Λ_1 and $\Lambda_{\frac{3}{2}}$ are necessary if one uses string fields in the large Hilbert space. One would now like to find a nonlinear version of (2.1) which does not involve picture-changing operators. One can check that equation (2.1) generalizes to the following nonlinear equations of motion and gauge invariances:

$$\begin{aligned}
\widetilde{\eta}(e^{-\Phi}(Qe^{\Phi})) &= -(\widetilde{\eta}\Psi)^2\,,\\
Q(e^{\Phi}(\widetilde{\eta}\Psi)e^{-\Phi}) &= 0\,,\\
\delta e^{\Phi} &= e^{\Phi}(\widetilde{\eta}\Lambda_1 - \{\widetilde{\eta}\Psi, \Lambda_{1/2}\}) + (Q\Lambda_0)e^{\Phi}\,,\\
\delta\Psi &= \widetilde{\eta}\Lambda_{3/2} + [\Psi, \widetilde{\eta}\Lambda_1] + \{Q + e^{-\Phi}(Qe^{\Phi}), \Lambda_{1/2}\}\,.
\end{aligned} \tag{2.2}$$

[2]Unlike [7] where $j_{\text{ghost}} = cb + \partial\phi$, the ghost number current will be defined here as $j_{\text{ghost}} = cb + \eta\xi$. So (η, ξ) carry ghost number $(+1, -1)$ and picture $(-1, +1)$ while $e^{n\phi}$ carries ghost number zero and picture n. This definition of ghost number agrees with that of [7] at zero picture, but has the advantage of commuting with picture-changing and spacetime supersymmetry.

[3]To avoid confusion with the picture subscript, the notation $\widetilde{\eta}$ will be used instead of η_0.

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Although these equations of motion and gauge invariances appear complicated, they simplify when expressed in terms of the ghost number one string fields $V = e^{-\Phi}(Qe^{\Phi})$ and $\Omega = \widetilde{\eta}\Psi$. In terms of V and Ω, (2.2) implies

$$\begin{aligned} QV = -V^2\,, \qquad Q\Omega = -\{\Omega, V\}\,, \qquad \widetilde{\eta}V = -\Omega^2\,, \qquad \widetilde{\eta}\Omega = 0\,, \\ \delta V = [Q+V, \widetilde{\eta}\Lambda_1 - \{\Omega, \Lambda_{1/2}\}]\,, \qquad \delta\Omega = \widetilde{\eta}(\{Q+V, \Lambda_{1/2}\} - \{\Omega, \Lambda_1\})\,. \end{aligned} \quad (2.3)$$

Note that the linearized contributions to (2.3) are the standard BRST conditions for ghost number one vertex operators in the small Hilbert space, i.e.

$$QV = \widetilde{\eta}V = Q\Omega = \widetilde{\eta}\Omega = 0\,, \qquad \delta V = Q\widetilde{\eta}\Lambda_1\,, \qquad \delta\Omega = \widetilde{\eta}Q\Lambda_{1/2}\,. \quad (2.4)$$

The above equations of motion and gauge invariances can be simplified even further by defining

$$G = G_0 + G_{-1} = Q + \widetilde{\eta} \qquad \text{and} \qquad A = A_0 + A_{-1/2} = V + \Omega = e^{-\Phi}(Qe^{\Phi}) + \widetilde{\eta}\Psi\,, \quad (2.5)$$

where G_n and A_n carry picture n. Then (2.2) and (2.3) are equivalent to

$$(G+A)^2 = 0, \quad \delta A = G\sigma + [A, \sigma]\,, \quad (2.6)$$

where $\sigma = \sigma_0 + \sigma_{1/2} = \widetilde{\eta}\Lambda_1 + \{Q+V, \Lambda_{1/2}\}$. By expanding (2.6) into the different pictures which contribute, one recovers equations (2.2) and (2.3). In other words, the four equations in the first line of (2.3) are implied by the contributions to $(G+A)^2 = 0$ at picture $[0, -1/2, -1, -3/2]$. The gauge transformations parameterized by $\Lambda_{1/2}$ and Λ_1 transform A as in (2.6) while those parameterized by Λ_0 and $\Lambda_{3/2}$ leave A invariant.

2.2 Problems with an action

Although the equations of (2.6) are an obvious analog of the bosonic string field theory equations of motion and gauge invariances, picture changing problems prevent the construction of an action which yields these equations of motion. If one sets to zero the Ramond string field, a Wess-Zumino-Witten-like action can be constructed out of the NS string field Φ which yields the desired equation of motion $\widetilde{\eta}(e^{-\Phi}(Qe^{\Phi})) = 0$. However, there is no way to consistently include a single Ramond string field Ψ into this action.

This impossibility can already be seen by analyzing the kinetic term for the Ramond string field. To recover the linearized equation of motion $Q\widetilde{\eta}\Psi = 0$, one would need a kinetic term $\langle \Psi Q \widetilde{\eta} \Psi \rangle$. However, since the large Hilbert space norm is defined as $\langle \xi e^{-2\phi} c\partial c \partial^2 c\rangle = 1$, a non-vanishing kinetic term must carry picture -1. But since Ψ cannot carry zero picture, $\langle \Psi Q\widetilde{\eta}\Psi\rangle$ cannot carry picture -1.

One possible solution [11, 2] would be to insert the picture-lowering operator $Y = c\partial\xi e^{-2\phi}$ at the midpoint of the kinetic term as

$$\langle \Psi Y(\pi) Q\widetilde{\eta}\Psi\rangle\,. \quad (2.7)$$

However, this would produce the equation of motion $Y(\pi)Q\widetilde{\eta}\Psi = 0$ and, since $Y(\pi)$

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has a nontrivial kernel, would not imply $Q\tilde{\eta}\Psi = 0$. Note that the kernel of $Y(\pi)$ only includes string states which are singular at their midpoint and therefore does not include "smooth" states, i.e. states constructed from the ground state with a finite number of mode operators. However, since the product of two smooth states is not necessarily smooth, the kernel of $Y(\pi)$ is nontrivial in the complete Fock space of string states.

One could try to truncate out all string states in the kernel of $Y(\pi)$ [12], but such a truncation would ruin the associativity properties of the midpoint interaction, leading to a breakdown of gauge invariance. In other words, if the operator T truncates out states in the kernel of $Y(\pi)$, the truncated product of three smooth string states $[A, B, C]$ is either $T(T(A \times B) \times C)$ or $T(A \times T(B \times C))$, which depends on the order of multiplication. One could also try to gauge away all states in the kernel of $Y(\pi)$ since the kinetic term of (2.7) is invariant under $\delta\Psi = \Lambda$ for $\Lambda \in \ker Y(\pi)$. However, there is no such gauge transformation which also leaves invariant the interaction terms. So just as insertion of the picture-raising operator $X = \{Q, \xi\}$ creates inconsistencies due to contact term divergences, insertion of the picture-lowering operator $Y = c\partial\xi e^{-2\phi}$ creates inconsistencies due to its nontrivial kernel.

Furthermore, the natural generalization of the R kinetic term of (2.7) to closed superstring field theory is the R-R kinetic term

$$\langle \Phi_{RR} Y \widehat{Y} (c - \widehat{c})_0 (Q + \widehat{Q}) \tilde{\eta} \tilde{\widehat{\eta}}\ \Phi_{RR} \rangle \tag{2.8}$$

where the unhatted and hatted operators are left and right-moving, and Φ_{RR} is the R-R closed string field at ghost number zero and (left,right)-moving picture $(1/2, 1/2)$. However, this kinetic term is not gauge invariant since $Y\widehat{Y}$ does not commute with $(b - \widehat{b})_0$. Recall that closed string fields Φ must satisfy the constraint $(b - \widehat{b})_0 \Phi = 0$ and the gauge transformation $\delta\Phi = (Q + \widehat{Q})(b - \widehat{b})_0 \Lambda$ only leaves the action invariant if $\{(b - \widehat{b})_0, [Q + \widehat{Q}, \mathcal{O}]\} = 0$ where $\mathcal{O}$ is the kinetic operator [13]. So (2.8) is inconsistent even before worrying about the nontrivial kernel of $Y\widehat{Y}$.

An alternative method for constructing a Ramond kinetic term is to split the Ramond states into two string fields, Ψ and $\overline{\Psi}$, where Ψ is defined to carry picture $+1/2$ and $\overline{\Psi}$ is defined to carry picture $-1/2$. Although this method necessarily breaks manifest $d = 10$ Lorentz covariance (since the sixteen component $d = 10$ spinor is broken into two eight component spinors), it allows one to construct the non-vanishing kinetic term $\langle \Psi Q \tilde{\eta} \overline{\Psi} \rangle$. As will be seen in the following sections, such a solution beautifully generalizes to a full nonlinear open superstring field theory action.

3. Open superstring formalism with three string fields

3.1 Equations of motion

Since one now has three string fields $[\Phi, \Psi, \overline{\Psi}]$, one has to decide how the NS and R states are distributed among these fields. Suppose that one can define a conserved

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charge C such that all superstring states carry one of three distinct values of this charge. Normalizing C such that η carries charge $C=-1$, it will be argued below that one can only construct consistent actions if these three distinct C-charges are $C=0$, $C=1/3$ and $C=-1/3$. States with charge $C=0$ will be represented by Φ, states with charge $C=1/3$ will be represented by Ψ, and states with charge $C=-1/3$ will be represented by $\overline{\Psi}$. The hermiticity properties of these three string fields will be discussed at the end of subsection (3.3).

So just as picture was used in the previous section to distribute states among two string fields, C-charge will be used here to distribute states among three string fields. As will be shown in section 4, different choices for C produce different actions with different manifest symmetries. Although there will be no $d=10$ Lorentz invariant choice of C, there are choices which preserve either $d=8$ Lorentz invariance or $N=1$ $d=4$ super-Poincaré invariance.

To construct an action in terms of the three string fields $[\Phi,\Psi,\overline{\Psi}]$, one first needs to find nonlinear equations of motion and gauge invariances which generalize the linearized equations

$$Q\widetilde{\eta}\Phi = Q\widetilde{\eta}\Psi = Q\widetilde{\eta}\overline{\Psi} = 0\,,$$
$$\delta\Phi = Q\Lambda_0+\widetilde{\eta}\Lambda_1\,,\qquad \delta\Psi = Q\Lambda_{1/3}+\widetilde{\eta}\Lambda_{4/3}\,,\qquad \delta\overline{\Psi} = Q\Lambda_{-1/3}+\widetilde{\eta}\Lambda_{2/3}\,, \quad (3.1)$$

where Λ_n carries C-charge n. To simplify the discussion, Q has temporarily been assumed to carry zero C-charge although this assumption will later be relaxed in subsection 3.3.

Following the discussion of the previous section, one would like to define a gauge field A in terms of $[\Phi,\Psi,\overline{\Psi}]$ such that $(G+A)^2=0$ gives the nonlinear equations of motion. Defining

$$G = G_0+G_{-1} = Q+\widetilde{\eta},\quad A = A_0+A_{-1/3}+A_{-2/3} = e^{-\Phi}(Qe^{\Phi})+e^{-\Phi}(Q\overline{\Psi})e^{\Phi}+\widetilde{\eta}\Psi\,, \quad (3.2)$$

one finds that the $[0,-1/3,-2/3,-1,-4/3,-5/3]$ C-charge contribution to $(G+A)^2=0$ implies

$$\begin{aligned} QA_0 &= -A_0^2\,, & \{Q+A_0,A_{-1/3}\} &= 0\,, & \{Q+A_0,A_{-2/3}\} &= -A^2_{-2/3}\,,\\ \widetilde{\eta}A_0 &= -\{A_{-2/3},A_{-1/3}\}\,, & \widetilde{\eta}A_{-1/3} &= -A^2_{-2/3}\,, & \widetilde{\eta}A_{-2/3} &= 0\,. \end{aligned} \quad (3.3)$$

The $[0,-1/3,-5/3]$ C-charge equations are automatically satisfied by (3.2), while the $[-2/3,-1,-4/3]$ C-charge equations imply the equations of motion

$$\begin{aligned} \widetilde{\eta}(e^{-\Phi}(Qe^{\Phi})) &= -\{\widetilde{\eta}\Psi, e^{-\Phi}(Q\overline{\Psi})e^{\Phi}\},\\ Q(e^{\Phi}(\widetilde{\eta}\Psi)e^{-\Phi}) &= -(Q\overline{\Psi})^2\,,\qquad \widetilde{\eta}(e^{-\Phi}(Q\overline{\Psi})e^{\Phi}) = -(\widetilde{\eta}\Psi)^2\,. \end{aligned} \quad (3.4)$$

Furthermore, the equation $(G+A)^2=0$ is invariant under the gauge transformation $\delta A = G\sigma+[A,\sigma]$ where

$$\sigma = \sigma_{1/3}+\sigma_0+\sigma_{-1/3} = \{Q+A_0,\Lambda_{1/3}\}+\widetilde{\eta}\Lambda_1+\{A_{-2/3},\Lambda_{2/3}\}+\widetilde{\eta}\Lambda_{2/3}\,. \quad (3.5)$$

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In terms of $[\Phi, \Psi, \overline{\Psi}]$, the gauge transformations are

$$\begin{aligned}
\delta e^{\Phi} &= e^{\Phi}(\widetilde{\eta}\Lambda_1 + \{A_{-2/3}, \Lambda_{2/3}\} - \{A_{-1/3}, \Lambda_{1/3}\}) + (Q\Lambda_0)e^{\Phi}\,, \\
\delta\Psi &= \{Q + A_0, \Lambda_{1/3}\} - \{A_{-1/3}, \Lambda_{2/3}\} - \{A_{-2/3}, \Lambda_1\} + \widetilde{\eta}\Lambda_{4/3}\,, \\
\delta\overline{\Psi} &= e^{\Phi}(\widetilde{\eta}\Lambda_{2/3} - \{A_{-2/3}, \Lambda_{1/3}\})e^{-\Phi} + \{Q\overline{\Psi}, \Lambda_0\} + Q\Lambda_{-1/3}\,.
\end{aligned} \tag{3.6}$$

So the linearized contribution to (3.4) and (3.6) reproduces (3.1).

Note that if $[\Phi, \Psi, \overline{\Psi}]$ did not have C-charges $[0, 1/3, -1/3]$, the equations implied by $(G + A)^2 = 0$ would be inconsistent. Firstly, Φ must have vanishing C-charge for e^{Φ} to have well-defined C-charge. And secondly, for $(Q\overline{\Psi})^2$ and $(\widetilde{\eta}\Psi)^2$ to have the same C-charge as $Q(e^{\Phi}(\widetilde{\eta}\Psi)e^{-\Phi})$ and $\widetilde{\eta}(e^{-\Phi}(Q\overline{\Psi})e^{\Phi})$ in (3.4), Ψ and $\overline{\Psi}$ must have C-charge $1/3$ and $-1/3$. Also note that with more than three string fields, $(G + A)^2 = 0$ would imply inconsistent equations of motion. For example, with four string fields, $(G + A)^2 = 0$ would imply equations with C-charges $[0, -1/4, -1/2, -3/4, -1, -5/4, -3/2, -7/4]$. As in (3.3), three of these equations could be satisfied by suitably defining A_n. However, this would leave five equations of motion for the four string fields.

3.2 Open superstring field theory action

To obtain the equations of motion of (3.4) from varying $[\Phi, \Psi, \overline{\Psi}]$ in an action, the $[-1, -2/3, -4/3]$ C-charge of these equations plus the $[0, -1/3, 1/3]$ C-charge of the string fields must equal the background C-charge. In other words, the nonvanishing norm $\langle \xi e^{-2\phi} c\partial c\partial^2 c\rangle$ must carry -1 C-charge. With this assumption, one can easily check that (3.4) comes from varying the action

$$\begin{aligned}
S = \Big\langle (e^{-\Phi}\widetilde{\eta}e^{\Phi})(e^{-\Phi}Qe^{\Phi}) + \int_0^1 dt(e^{-\widehat{\Phi}}\partial_t e^{\widehat{\Phi}})\{e^{-\widehat{\Phi}}\widetilde{\eta}e^{\widehat{\Phi}}, e^{-\widehat{\Phi}}Qe^{\widehat{\Phi}}\} + \\
+ e^{-\Phi}(Q\overline{\Psi})e^{\Phi}(\widetilde{\eta}\Psi) - \frac{1}{3}\overline{\Psi}(Q\overline{\Psi})^2 + \frac{1}{3}\Psi(\widetilde{\eta}\Psi)^2 \Big\rangle\,,
\end{aligned} \tag{3.7}$$

where $\widehat{\Phi}(t)$ is a function defined for $0 \le t \le 1$ which satisfies $\widehat{\Phi}(0) = 0$ and $\widehat{\Phi}(1) = \Phi$.

The first line of (3.7) is the same WZW-like action constructed in [6] for the NS sector. To give an interpretation for the second line of (3.7), note that the small Hilbert space norm

$$\langle \widetilde{\eta}(\xi e^{-2\phi} c\partial c\partial^2 c)\rangle = \langle e^{-2\phi} c\partial c\partial^2 c\rangle = 1 \tag{3.8}$$

can be used when all fields in the correlation function are annihilated by the η zero mode [7]. Likewise, one can define a different small Hilbert space norm

$$\langle Q(\xi e^{-2\phi} c\partial c\partial^2 c)\rangle = \langle 2\eta c\partial c\rangle = 1 \tag{3.9}$$

when all fields in the correlation function are annihilated by Q.

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These two small Hilbert spaces resemble chiral and antichiral superspaces where the norm of (3.8) is used for chiral F-terms and the norm of (3.9) is used for antichiral F-terms. So it is natural to define a "chiral" field Ω as any field satisfying $\widetilde{\eta}\Omega = 0$, and an "antichiral" field $\overline{\Omega}$ as any field satisfying $Q\overline{\Omega} = 0$. To distinguish the different Hilbert space norms, the notation $\langle\ \rangle_F$ and $\langle\ \rangle_{\overline{F}}$ will denote the small Hilbert space norms of (3.8) and (3.9) respectively, and the notation $\langle\ \rangle_D$ will denote the large Hilbert space norm.

Since $\langle\Psi(\widetilde{\eta}\Psi)^2\rangle_D = \langle(\widetilde{\eta}\Psi)^3\rangle_F$ and $\langle\overline{\Psi}(Q\overline{\Psi})^2\rangle_D = \langle(Q\overline{\Psi})^3\rangle_{\overline{F}}$, the second line of (3.7) can be written as

$$\langle e^{-\Phi}\overline{\Omega}e^{\Phi}\Omega\rangle_D - \frac{1}{3}\langle\overline{\Omega}^3\rangle_{\overline{F}} + \frac{1}{3}\langle\Omega^3\rangle_F\,, \tag{3.10}$$

where $\Omega = \widetilde{\eta}\Psi$ is a chiral string field and $\overline{\Omega} = Q\overline{\Psi}$ is an antichiral string field. So the second line of (3.7) can be interpreted as the standard kinetic term and Yukawa potential for chiral and antichiral fields. Note that the cohomologies of $\widetilde{\eta}$ and Q are trivial in the large Hilbert space, so any chiral superfield Ω can be written as $\widetilde{\eta}\Psi$ for some Ψ and any antichiral superfield $\overline{\Omega}$ can be written as $Q\overline{\Psi}$ for some $\overline{\Psi}$. One can therefore treat Ω and $\overline{\Omega}$ as fundamental chiral and antichiral string fields in the action and forget about Ψ and $\overline{\Psi}$. Since $\langle\Omega^3\rangle_F$ cannot be written as a D-term without introducing Ψ, one expects for the usual reasons that this F-term does not receive quantum corrections.

3.3 Q with nonzero C-charge

Although the construction of (3.7) assumed that Q carries zero C-charge, this assumption can be slightly relaxed. To preserve the structure of the equations implied by $(G+A)^2 = 0$, it will be necessary to assume only that $G = G_0 + G_{-1/3} + G_{-2/3} + G_{-1}$. In other words, it will be assumed that Q and $\widetilde{\eta}$ only contain terms carrying C-charge $[0, -1/3, -2/3, -1]$. Note that $(Q+\widetilde{\eta})^2 = 0$ implies that $G_0^2 = G_{-1}^2 = 0$, and it will also be assumed that G_0 and G_{-1} have trivial cohomology in the large Hilbert space. With this assumption, a chiral string field Ω and antichiral string field $\overline{\Omega}$ can be defined by $G_{-1}\Omega = 0$ and $G_0\overline{\Omega} = 0$, which implies that $\Omega = G_{-1}\Psi$ and $\overline{\Omega} = G_0\overline{\Psi}$ for some Ψ and $\overline{\Psi}$.

One can check that $(G+A)^2 = 0$ and $\delta A = G\sigma + [A, \sigma]$ imply consistent equations of motion and gauge invariances where

$$\begin{aligned} &A_0 = e^{-\Phi}(G_0 e^{\Phi})\,, \qquad A_{-1/3} = e^{-\Phi}(G_{-1/3}e^{\Phi}) + e^{-\Phi}\overline{\Omega}e^{\Phi}\,, \qquad A_{-2/3} = \Omega\,, \\ &\sigma_{1/3} = \{G_0 + A_0, \Lambda_{1/3}\}\,, \qquad \sigma_0 = G_{-1}\Lambda_1 + \{G_{-2/3} + \Omega, \Lambda_{2/3}\}, \quad \sigma_{-1/3} = G_{-1}\Lambda_{2/3}\,. \end{aligned} \tag{3.11}$$

Defining $\mathcal{G}_n = G_n + A_n$, the equations of motion and gauge invariances of (3.4) and (3.6) generalize to

$$\{G_{-1}, \mathcal{G}_0\} = -\{\mathcal{G}_{-2/3}, \mathcal{G}_{-1/3}\}\,,$$

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$$
\begin{aligned}
\{\mathcal{G}_0, \mathcal{G}_{-2/3}\} &= -(\mathcal{G}_{-1/3})^2 \,, \\
\{G_{-1}, \mathcal{G}_{-1/3}\} &= -(\mathcal{G}_{-2/3})^2 \,,
\end{aligned}
\tag{3.12}
$$

and

$$
\begin{aligned}
\delta e^{\Phi} &= e^{\Phi}(G_{-1}\Lambda_1 + \mathcal{G}_{-2/3}\Lambda_{2/3} - \mathcal{G}_{-1/3}\Lambda_{1/3}) + (G_0\Lambda_0)e^{\Phi} \,, \\
\delta\Psi &= \mathcal{G}_0\Lambda_{1/3} - \mathcal{G}_{-2/3}\Lambda_1 - \mathcal{G}_{-1/3}\Lambda_{2/3} + G_{-1}\Lambda_{4/3} \,, \\
\delta\overline{\Psi} &= e^{\Phi}(G_{-1}\Lambda_{2/3} - \mathcal{G}_{-2/3}\Lambda_{1/3})e^{-\Phi} + \{G_{-1/3} + \overline{\Omega}, \Lambda_0\} + G_0\Lambda_{-1/3} \,.
\end{aligned}
\tag{3.13}
$$

The action which produces the equations of motion of (3.12) is [5]:

$$
\begin{aligned}
S = \Big\langle & (e^{-\Phi}G_{-1}e^{\Phi})(e^{-\Phi}G_0e^{\Phi}) + (e^{-\Phi}G_{-2/3}e^{\Phi})(e^{-\Phi}G_{-1/3}e^{\Phi}) + \\
& + \int_0^1 dt(e^{-\widehat{\Phi}}\partial_t e^{\widehat{\Phi}}) \Big(\{e^{-\widehat{\Phi}}G_{-1}e^{\widehat{\Phi}}, e^{-\widehat{\Phi}}G_0e^{\widehat{\Phi}}\} + \{e^{-\widehat{\Phi}}G_{-2/3}e^{\widehat{\Phi}}, e^{-\widehat{\Phi}}G_{-1/3}e^{\widehat{\Phi}}\}\Big) + \\
& + e^{-\Phi}\overline{\Omega}e^{\Phi}\Omega + \overline{\Omega}e^{\Phi}(G_{-2/3}e^{-\Phi}) - \Omega e^{-\Phi}(G_{-1/3}e^{\Phi})\Big\rangle_D - \\
& - \left\langle \frac{1}{2}\overline{\Omega}G_{-1/3}\overline{\Omega} + \frac{1}{3}\overline{\Omega}^3 \right\rangle_{\overline{F}} + \left\langle \frac{1}{2}\Omega G_{-2/3}\Omega + \frac{1}{3}\Omega^3 \right\rangle_F \,,
\end{aligned}
\tag{3.14}
$$

where $\langle\ \rangle_F$ and $\langle\ \rangle_{\overline{F}}$ are defined using the Hilbert space norms $\langle G_{-1}(\xi e^{-2\phi}c\partial c\partial^2 c)\rangle_F = 1$ and $\langle G_0(\xi e^{-2\phi}c\partial c\partial^2 c)\rangle_{\overline{F}} = 1$. Note that the chiral and antichiral F-terms in the last line of (3.14) remember holomorphic and antiholomorphic Chern-Simons terms and are not expected to receive quantum corrections.

There are two possible definitions of hermiticity which are consistent with the action of (3.14). The first possibility is that all string fields $[\Phi, \Psi, \overline{\Psi}]$ and operators G_n are independently hermitian. The second possibility is that Φ is antihermitian, $\overline{\Psi}$ is the hermitian conjugate of Ψ, and G_n is the hermitian conjugate of G_{-1-n}.[4]

4. Splitting the states into three string fields

4.1 Conditions for the C-charge

In this section, the action of (3.14) will be made explicit by giving two examples of C-charge. As discussed in section 3, consistency of the action implies that the C-charge must be a conserved charge with the following properties: 1) All superstring states must carry C-charge 0 or $\pm 1/3$; 2) All terms in $Q + \widetilde{\eta}$ must carry C-charge $[0, -1/3, -2/3, -1]$ where the terms with 0 and -1 C-charge have trivial cohomology; and 3) The large Hilbert space background charge $\xi e^{-2\phi}c\partial c\partial^2 c$ must carry C-charge -1.

[4]Using the second hermiticity definition, the action of (3.14) naively appears to be imaginary. However, in the explicit example considered in subsection 4.3, $\xi e^{-2\phi}c\partial c\partial^2 c$ will be imaginary with this definition. So if one defines $\langle \xi e^{-2\phi}c\partial c\partial^2 c\rangle = 1$, the action of (3.14) is real since the norm is imaginary.

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Since the term $\eta\partial\eta e^{2\phi}b$ in Q is the term with trivial cohomology, this term should carry zero C-charge. And since both η and $\xi e^{-2\phi}c\partial c\partial^2 c$ must carry C-charge -1, $e^{n\phi}$ must carry C-charge n and (b,c) must carry C-charge zero. This implies that $C = P + \frac{1}{3}N$ where P is picture and N is some conserved charge constructed from the RNS matter fields. Furthermore, since $3C$ must be an integer, N must be chosen such that NS states carry integer N-charge and R states carry half-integer N-charge.

In a flat background, examples of such N-charges are

$$N = \sum_{j=1}^{J} \int \psi^{2j-2}\psi^{2j-1} \tag{4.1}$$

for $J = 1$, $J = 3$ or $J = 5$. These examples (up to Wick rotations) manifestly preserve an $\mathrm{SO}(10-2J) \times \mathrm{U}(J)$ subgroup of the $d = 10$ Lorentz group. The examples $J = 1$ and $J = 3$ will be explicitly discussed below, and the example $J = 5$ can be treated similarly if one ignores hermiticity questions.

Note that J must be odd in (4.1) in order that R states carry half-integer N-charge. This dependence on J might seem strange, but at the end of section 5, an R-R kinetic term will be constructed with $d = 10 - 2J$ Lorentz invariance. When J is even, the type-IIB R-R spectrum in $d = 10 - 2J$ contains a self-dual $(5 - J)$-form field strength, so one expects to find problems with constructing an action.

4.2 Manifest $d = 8$ Lorentz covariance

Splitting the Ramond states into different string fields implies that the sixteen component $d = 10$ spinor must split into two eight component spinors. So the maximum Lorentz subgroup which can be manifestly preserved is $d = 8$ Lorentz covariance. As will now be shown, this can be achieved by defining

$$C = P + \frac{1}{3}\int \psi^0\psi^9 \tag{4.2}$$

where P is the picture and $\int \psi^0\psi^9$ is the $\mathrm{SO}(1,1)$ charge in the M_{09} direction. An $\mathrm{SO}(1,1)$ boost direction has been chosen, so following the discussion at the end of section 3, one can use the first hermiticity definition in which Ψ and $\overline{\Psi}$ are independent hermitian string fields. If one had instead chosen a $\mathrm{U}(1)$ rotation direction (e.g. $C = P + \frac{i}{3}\int \psi^1\psi^2$), one would use the second hermiticity definition in which Ψ and $\overline{\Psi}$ are hermitian conjugate string fields.

With the choice of (4.2), $Q + \widetilde{\eta}$ splits into terms of C-charge $[0, 1/3, -1/3, -1]$ where the terms of C-charge $\pm 1/3$ are $\eta e^{\phi}\partial x^-\psi^+$ and $\eta e^{\phi}\partial x^+\psi^-$ using the notation $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^9)$ and $\psi^\pm = \frac{1}{\sqrt{2}}(\psi^0 \pm \psi^9)$. To remove the unwanted term of C-charge $+1/3$, one can perform the similarity transformation

$$Q + \widetilde{\eta} \to e^R(Q + \widetilde{\eta})e^{-R} \qquad \text{where} \qquad R = \int c\xi e^{-\phi}\psi^+\partial x^- \,. \tag{4.3}$$

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To show that (4.3) only has terms with C-charge $[0,-1/3,-2/3,-1]$, it is convenient to use the result of [14] where the RNS BRST operator was written as $Q=e^{-S}(-\int\eta\partial\eta e^{2\phi}b)e^{S}$ with

$$S=\int\left(c\xi e^{-\phi}\psi^m\partial x_m+\frac{1}{2}\partial\phi c\partial c\xi\partial\xi e^{-2\phi}\right)\qquad\text{and}\qquad m=0\text{ to }9\,.\tag{4.4}$$

So the similarity transformation of (4.3) takes $Q+\widetilde{\eta}$ into

$$\begin{aligned}e^{R}(Q+\widetilde{\eta})e^{-R}&=e^{R}e^{-S}\left(-\int\eta\partial\eta e^{2\phi}b\right)e^{S}e^{-R}+e^{R}\widetilde{\eta}e^{-R}\\&=e^{-U}\left(-\int\eta\partial\eta e^{2\phi}b\right)e^{U}+\int\eta e^{\phi}\psi^-\partial x^+ +\int ce^{-\phi}\psi^+\partial x^- +\widetilde{\eta}\,,\end{aligned}\tag{4.5}$$

where

$$U=\int\left(c\xi e^{-\phi}\psi^k\partial x_k+\frac{1}{2}(\partial\phi+\psi^0\psi^9)c\partial c\xi\partial\xi e^{-2\phi}\right)\qquad\text{and }k=1\text{ to }8\,.\tag{4.6}$$

So after performing the similarity transformation of (4.6), $G=e^{R}(Q+\widetilde{\eta})e^{-R}$ only contains terms of C-charge $[0,-1/3,-2/3,-1]$ which are given by

$$\begin{aligned}G_0&=e^{-U}\left(-\int\eta\partial\eta e^{2\phi}b\right)e^{U}\,, &\qquad G_{-1/3}&=\int\eta e^{\phi}\psi^-\partial x^+\,,\\G_{-2/3}&=\int ce^{-\phi}\psi^+\partial x^-\,, & G_{-1}&=\textstyle\int\eta\,.\end{aligned}\tag{4.7}$$

With G_n defined by (4.7) and the string fields $[\Phi,\Psi,\overline{\Psi}]$ defined using (4.2), (3.14) gives a manifestly $d=8$ Lorentz covariant open superstring field theory action.

Note that (4.2) implies that the massless gluon $A_m(x)$ and gluino $\chi^\alpha(x)$ split into the following components of the string fields:

$$\begin{aligned}\Phi&=\xi ce^{-\phi}\psi^k A_k(x)+\cdots\,,\qquad \Psi=\xi ce^{-\phi}\psi^+A_+(x)+\xi ce^{-\frac{1}{2}\phi}\Sigma_a\chi^a(x)+\cdots\,,\\\overline{\Psi}&=\xi ce^{-\phi}\psi^-A_-(x)+\xi\partial\xi c\partial ce^{-\frac{5}{2}\phi}\Sigma_{\dot a}\overline{\chi}^{\dot a}(x)+\cdots\,,\end{aligned}\tag{4.8}$$

where $\chi^a=(\gamma^+\chi)^a$ and $\overline{\chi}^{\dot a}=(\gamma^-\chi)^{\dot a}$ are the SO(8) components of χ^α. One can check that the gluino contribution

$$\begin{aligned}S&=\frac{1}{2}\,\mathrm{Tr}\int d^{10}x\chi^\alpha\gamma^m_{\alpha\beta}(\partial_m\chi^\beta+[A_m,\chi^\beta])\\&=\mathrm{Tr}\int d^{10}x\left(\chi^a\sigma^k_{a\dot b}(\partial_k\overline{\chi}^{\dot b}+[A_k,\overline{\chi}^{\dot b}]\right)+\frac{1}{2}\overline{\chi}^{\dot a}(\partial_-\overline{\chi}^{\dot a}+[A_-,\overline{\chi}^{\dot a}])+\\&\qquad+\frac{1}{2}\chi^a(\partial_+\chi^a+[A_+,\chi^a]))\end{aligned}\tag{4.9}$$

comes from the terms

$$\langle e^{-\Phi}\overline{\Omega}e^{\Phi}\Omega\rangle_D-\left\langle\frac{1}{2}\overline{\Omega}G_{-1/3}\overline{\Omega}+\frac{1}{3}\overline{\Omega}^3\right\rangle_{\overline{F}}+\left\langle\frac{1}{2}\Omega G_{-2/3}\Omega+\frac{1}{3}\Omega^3\right\rangle_F\tag{4.10}$$

in (3.14).

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4.3 Manifest $N = 1$ $d = 4$ super-Poincaré covariance

A second possible choice for the C-charge is

$$C = P + \frac{i}{3} \int (\psi^4 \psi^5 + \psi^6 \psi^7 + \psi^8 \psi^9) \,. \tag{4.11}$$

This charge easily generalizes to $C = P + \frac{1}{3} \int \partial H$ for compactification on a Calabi-Yau threefold with U(1) current $J_{CY} = \partial H$. Because ∂H is antihermitian, the string fields and operators must satisfy the second hermiticity definition, i.e.

$$\Phi^\dagger = -\Phi \,, \qquad \Psi^\dagger = \overline{\Psi} \,, \qquad G_0^\dagger = G_{-1} \,, \qquad G_{-1/3}^\dagger = G_{-2/3} \,. \tag{4.12}$$

As will now be shown, the above hermiticity conditions are natural if one rewrites the RNS worldsheet variables in terms of $d = 4$ Green-Schwarz-like variables [15], which also allows $N = 1$ $d = 4$ super-Poincaré covariance to be made manifest.

The first step to constructing an $N = 1$ $d = 4$ super-Poincaré covariant action is to perform the similarity transformation

$$Q + \widetilde{\eta} \to e^{R + \frac{1}{2}U} (Q + \widetilde{\eta}) e^{-R - \frac{1}{2}U} \tag{4.13}$$

where

$$U = \int \left(c\xi e^{-\phi} \psi^p \partial x_p + \frac{1}{2} (\partial\phi + \partial H) c \partial c \xi \partial \xi e^{-2\phi} \right) , \qquad R = \int c \xi e^{-\phi} \psi^{+j} \partial x^{-j} \,,$$

$p = 0$ to 3, $j = 1$ to 3, $\psi^{\pm j} = \frac{1}{\sqrt{2}} (\psi^{2j+2} \pm i \psi^{2j+3})$ and $x^{\pm j} = \frac{1}{\sqrt{2}} (x^{2j+2} \pm i x^{2j+3})$. Since $Q = e^{-S} (-\int \eta \partial \eta e^{2\phi} b) e^S$ where S is defined in (4.4), one finds

$$\begin{aligned} e^{R + \frac{1}{2}U} (Q + \widetilde{\eta}) e^{-R - \frac{1}{2}U} &= e^{-\frac{1}{2}U} \left(-\int \eta \partial \eta e^{2\phi} b \right) e^{\frac{1}{2}U} + \int \eta e^{\phi} \psi^{-j} \partial x^{+j} + \\ &\quad + \int c e^{-\phi} \psi^{+j} \partial x^{-j} + e^{\frac{1}{2}U} \widetilde{\eta} e^{-\frac{1}{2}U} \,. \end{aligned} \tag{4.14}$$

Equation (4.14) can be written in manifestly $N = 1$ $d = 4$ super-Poincaré covariant notation by defining the $d = 4$ Green-Schwarz-like variables [15]

$$\begin{aligned} \theta^\alpha = e^{\frac{1}{2}\phi} \Sigma^\alpha e^{-\frac{1}{2}H} \,, \quad \overline{\theta}^{\dot\alpha} = c \xi e^{-\frac{3}{2}\phi} \Sigma^{\dot\alpha} e^{\frac{1}{2}H} \,, \quad p_\alpha = e^{-\frac{1}{2}\phi} \Sigma_\alpha e^{\frac{1}{2}H} \,, \quad \overline{p}_{\dot\alpha} = b \eta e^{\frac{3}{2}\phi} \Sigma_{\dot\alpha} e^{-\frac{1}{2}H} \,, \\ \partial\rho = 3\phi + cb + 2\xi\eta - \partial H \,, \quad \Gamma^{+j} = \xi e^{-\phi} \psi^{+j} \,, \quad \Gamma^{-j} = \eta e^{\phi} \psi^{-j} \,, \end{aligned} \tag{4.15}$$

where Σ^α and $\Sigma^{\dot\alpha}$ are $d = 4$ spin fields constructed from $[\psi^0, \psi^1, \psi^2, \psi^3]$ and $(\alpha, \dot\alpha) = 1$ to 2. Note that the variables of (4.15) are GSO-projected and satisfy free-field OPE's.

In terms of these variables, one can check that

$$\begin{aligned} e^{R + \frac{1}{2}U} (Q + \widetilde{\eta}) e^{-R - \frac{1}{2}U} = \int \Bigg[&\frac{1}{2} d^\alpha d_\alpha e^\rho + \Gamma^{-j} \partial x^{+j} + \frac{1}{2} \epsilon^{jkl} \partial x^{-j} \Gamma^{-k} \Gamma^{-l} e^{-\rho} + \\ &+ \frac{1}{12} \overline{d}^{\dot\alpha} \overline{d}_{\dot\alpha} e^{-2\rho} \epsilon^{jkl} \Gamma^{-j} \Gamma^{-k} \Gamma^{-l} \Bigg] \,, \end{aligned} \tag{4.16}$$

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where $d_\alpha = p_\alpha + \frac{i}{2}\overline{\theta}^{\dot\alpha}\partial x_p \sigma^p_{\alpha\dot\alpha} - \frac{1}{4}(\overline{\theta})^2\partial\theta_\alpha + \frac{1}{8}\theta_\alpha\partial(\overline{\theta})^2$ and $\overline{d}_{\dot\alpha} = \overline{p}_{\dot\alpha} + \frac{i}{2}\theta^\alpha\partial x_p\sigma^p_{\alpha\dot\alpha} - \frac{1}{4}(\theta)^2\partial\overline{\theta}_{\dot\alpha} + \frac{1}{8}\overline{\theta}_{\dot\alpha}\partial(\theta)^2$ are supersymmetric combinations of the fermionic momenta. Since $C = 1/3\int\Gamma^{+j}\Gamma^{-j}$, (4.16) implies that

$$\begin{aligned} G_0 &= \int \frac{1}{2} d^\alpha d_\alpha e^\rho\,, & G_{-1/3} &= \int \Gamma^{-j}\partial x^{+j}\,, \\ G_{-2/3} &= \int \frac{1}{2}\epsilon^{jkl}\partial x^{-j}\Gamma^{-k}\Gamma^{-l}e^{-\rho}\,, & G_{-1} &= \int \frac{1}{12}\overline{d}^{\dot\alpha}\overline{d}_{\dot\alpha}e^{-2\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}\,. \end{aligned} \tag{4.17}$$

The hermiticity properties of (4.12) are satisfied if one defines

$$\begin{aligned} \theta^\dagger_\alpha = \overline{\theta}_{\dot\alpha}\,, \qquad p^\dagger_\alpha = -\overline{p}_{\dot\alpha}\,, \qquad (\Gamma^{-j})^\dagger &= \frac{1}{2}\epsilon^{jkl}\Gamma^{-k}\Gamma^{-l}e^{-\rho}\,, \\ (\Gamma^{+j})^\dagger = \frac{1}{2}\epsilon^{jkl}\Gamma^{+k}\Gamma^{+l}e^{\rho}\,, \qquad\qquad (e^\rho)^\dagger &= \frac{1}{6}e^{-2\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}\,. \end{aligned} \tag{4.18}$$

Note that $\xi e^{-2\phi}c\partial c\partial^2 c = \frac{1}{24}(\theta)^2(\overline{\theta})^2 e^{-\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}$ is imaginary, as discussed in footnote 4. Since $j_{\text{ghost}} = \partial\rho + \Gamma^{-j}\Gamma^{+j}$, the ghost-number zero string fields $[\Phi, \Psi, \overline{\Psi}]$ carry ρ-charge $[0, +1, -1]$.

So using the operators of (4.17) in the action of (3.14), one gets a manifestly $N = 1$ $d = 4$ super-Poincaré covariant open superstring field theory action. As shown in [5], the massless contribution to this action reproduces the $d = 10$ super-Yang-Mills action written in terms of $N = 1$ $d = 4$ superfields [16] where the D-terms and F-terms in (3.14) reproduce the standard $N = 1$ $d = 4$ superspace D-terms and F-terms.

5. Open questions

In this paper, it was argued that construction of a consistent open superstring field theory action requires splitting the superstring states into three string fields which carry conserved C-charge 0 and $\pm 1/3$. Different choices for splitting the superstring states produce different actions with different manifest symmetries. This construction raises several obvious questions.

One question is how to generalize the action of (3.14) to include the GSO($-$) superstring states which are present for non-BPS D-branes and for D-brane/anti-D-brane configurations [17]. As will be shown in a separate paper [18] with Carlos Tello Echevarria, these GSO($-$) states can be easily included by adjoining 2×2 matrices to the three string fields and operators of (3.14), as was done for the NS action in [19]. Such an action might be useful for studying broken supersymmetry before tachyon condensation as proposed by Yoneya [20].

A second question is if the different actions produced by different splittings are related by a field redefinition. It is easy to show that the equations of motion of (3.12) for different splittings are related by a field redefinition since (3.12) can be written

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in a splitting-independent manner as $(G+A)^2=0$. So any solution to the equations of motion using one splitting is also a solution using another splitting. However, it is not obvious that there exists an off-shell field redefinition which relates the different actions.

A third question is which worldsheet conformal field theory backgrounds allow construction of an open superstring field theory action, i.e. which backgrounds allow definition of a C-charge with the desired properties. It might seem strange that not all $N=1$ $c=15$ superconformal field theory (scft) backgrounds allow construction of an open superstring field theory action. However, this should not be too surprising since, for example, R-R backgrounds cannot be described by an $N=1$ $c=15$ scft since they mix the RNS matter and ghost fields.

To describe R-R backgrounds [21, 22], one needs to embed the superstring in a worldsheet $N=2$ $c=6$ scft [10]. An open superstring field theory action in a worldsheet $N=2$ scft background can be defined by replacing $G=Q+\widetilde{\eta}$ of (3.14) with $G=\int(G^+ + \widetilde{G}^+)$ where G^+ and $\widetilde{G}^+$ are constructed from the fermionic worldsheet $N=2$ generators as explained in [10]. For example, the action constructed in subsection (4.3) generalizes to an $AdS_2\times S^2$ background with R-R flux by replacing the flat $d=4$ Minkowski background with the $N=2$ scft described in [22]. It would be interesting to know precisely which $N=2$ $c=6$ scft backgrounds allow construction of an open superstring field theory action.

A final question is if the methods of this paper are useful for constructing a closed superstring field theory action. Although the NS-NS contribution to such an action can be constructed as in [23], the only successful construction up to now of a kinetic term for the R-R sector [24] uses the SU$(1,1)$ formalism [25] of Siegel and Zwiebach. However, even for bosonic string field theory, the SU$(1,1)$ formalism has not yet been generalized to include interactions. Furthermore, the R-R kinetic term of [24] involves an infinite number of fields, which is not surprising because of the self-dual five-form in the type-IIB R-R sector.

Since the closed string field can be understood as the "left-right" product of two open string fields, the methods of this paper suggest introducing a closed superstring field $\Phi_{m,\widehat{n}}$ carrying left-moving C-charge m for $m\in[0,\pm 1/3]$ and right-moving C-charge $\widehat{n}$ for $\widehat{n}\in[0,\pm 1/3]$. When Q carries zero C-charge, one can construct the closed superstring kinetic term

$$S_{\text{closed}} = \sum_{m,\widehat{n}=-1/3}^{1/3} \left\langle \Phi_{-m,-\widehat{n}}(c-\widehat{c})_0(Q+\widehat{Q})\widetilde{\eta}\widetilde{\widetilde{\eta}}\;\Phi_{m,\widehat{n}} \right\rangle, \tag{5.1}$$

which is the natural closed string generalization of the open superstring kinetic term $\langle \Phi Q\widetilde{\eta}\Phi\rangle$. Unlike the kinetic term of (2.8), (5.1) is gauge invariant when $\Phi_{m,\widehat{n}}$ satisfies the constraint $(b-\widehat{b})_0\Phi_{m,\widehat{n}}=0$. The kinetic term of (5.1) can be written in a more

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symmetric form as

$$S_{\text{closed}} = \sum_{m,\widehat{n}=-1/3}^{1/3} \left\langle \Phi_{-m,-\widehat{n}} (c\eta + \widehat{c}\widehat{\eta})_0 (Q + \widehat{Q})(\eta + \widehat{\eta})_0 \, \Phi_{m,\widehat{n}} \right\rangle \tag{5.2}$$

since the $(b - \widehat{b})_0$ constraint implies that only the $(c - \widehat{c})_0(\eta - \widehat{\eta})_0$ part of $(c\eta + \widehat{c}\widehat{\eta})_0$ contributes to the action. Note that (5.2) can be generalized to any N=(2,2) $c = 6$ scft as

$$S_{\text{closed}} = \sum_{m,\widehat{n}=-1/3}^{1/3} \left\langle \Phi_{-m,-\widehat{n}} (J^{++} + \widehat{J}^{++})_0 (G^{+} + \widehat{G}^{+})_0 (\widetilde{G}^{+} + \widehat{\widetilde{G}}^{+})_0 \Phi_{m,\widehat{n}} \right\rangle , \tag{5.3}$$

where $(G^+, \widetilde{G}^+, J^{++})$ and $(\widehat{G}^+, \widehat{\widetilde{G}}^+, \widehat{J}^{++})$ are constructed from the left and right-moving $N = 2$ superconformal generators as described in [10].

Unfortunately, the kinetic term of (5.1) does not seem to generalize when Q carries non-zero C-charge, i.e. when $G = Q+\widetilde{\eta} = G_0+G_{-1/3}+G_{-2/3}+G_{-1}$. Nevertheless, using the C-charges constructed in section 4, one could consider defining (5.1) where Q and $\widetilde{\eta}$ are replaced by G_0 and G_{-1}. Using the C-charge of subsection 4.2, this would give the eight-dimensional contribution to the kinetic term, i.e. the contribution from string fields which are independent of $x^{\pm}$ and $\psi^{\pm}$. And using the the C-charge of subsection 4.3, this would give the four-dimensional contribution to the kinetic term, i.e. the contribution from string fields which are independent of $x^{\pm j}$ and $\Gamma^{\pm j}$. Note that these four and eight-dimensional contributions do not contain self-dual field strengths in the R-R sector, so one does not expect any problems. However, if one could have constructed a C-charge which preserved $d = 10 - 2J$ Lorentz invariance for J even, one would expect problems since there are self-dual $(5 - J)$-form field strengths in the $(10 - 2J)$-dimensional contribution to the R-R kinetic term.

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Acknowledgments

I would like to thank Carlos Tello Echevarria, Cumrun Vafa and Barton Zwiebach for useful discussions, and CNPq grant 300256/94-9, Pronex grant 66.2002/1998-9, and FAPESP grant 99/12763-0 for partial financial support. This research was partially conducted during the period the author was employed by the Clay Mathematics Institute as a CMI Prize Fellow.

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